

# CHAPTER 12

## SHIP STABILITY AND BUOYANCY

**Learning Objectives:** Recall the terminology used for ship stability; the laws of physics and trigonometry used to determine stability and buoyancy of a ship; and the effects of buoyancy, gravity, and weight shifts on ship stability.

Under the guidance of the damage control assistant, damage control personnel provide the first line of defense to ensure your ship is as seaworthy as possible. Your responsibilities may include preparing daily draft reports, taking soundings, or perhaps you may stand watch operating a ballasting console.

In this chapter, you will be introduced to the laws of mathematics and physics used to determine the buoyancy and stability of a ship. Also, there are various engineering and mathematical principles that you will become familiar with as you study this chapter. Detailed information on these subjects is provided in the *Naval Ships' Technical Manual (NSTM)*, chapter 079, volume 1, and in *NSTM*, chapter 096. You can find additional information on these subjects in publications you will find listed in the *Damage Controlman Advancement Handbook*.

### PRINCIPLES OF STABILITY

**Learning Objectives:** Recall the basic functions of trigonometry, the terminology used for ship stability, the effects of buoyancy and gravity on ship stability, and the effects of weight shifts on ship stability.

To comprehend the principles of ship stability fully, you must have a basic understanding of trigonometry and the functions of right triangles. Generally speaking, the weight of a ship in the water is “pushing” straight down, and the seawater that it displaces is “pushing” straight back up. When no other forces are acting on the ship, all these forces cancel each other out and equilibrium exists. However, when the center of gravity moves from directly above the center of buoyancy, there is an “inclining moment.” When this occurs, this force is considered to be at right angles to the forces of gravity and buoyancy. An understanding of trigonometry is required to understand the effects and results of these actions.

### TRIGONOMETRY

Trigonometry is the study of triangles and the interrelationship of the sides and the angles of a triangle. In determining ship stability, only that part of trigonometry pertaining to right triangles is used. There is a fixed relationship between the angles of a right triangle and the ratios of the lengths of the sides of the triangle. These ratios are known as trigonometric functions and have been given the following names: sine, cosine, tangent, cotangent, secant, and cosecant. The three trigonometric functions required for ship stability work are the **sine**, **cosine**, and **tangent**. Figure 12-1 shows these trigonometric relations.

#### Sine

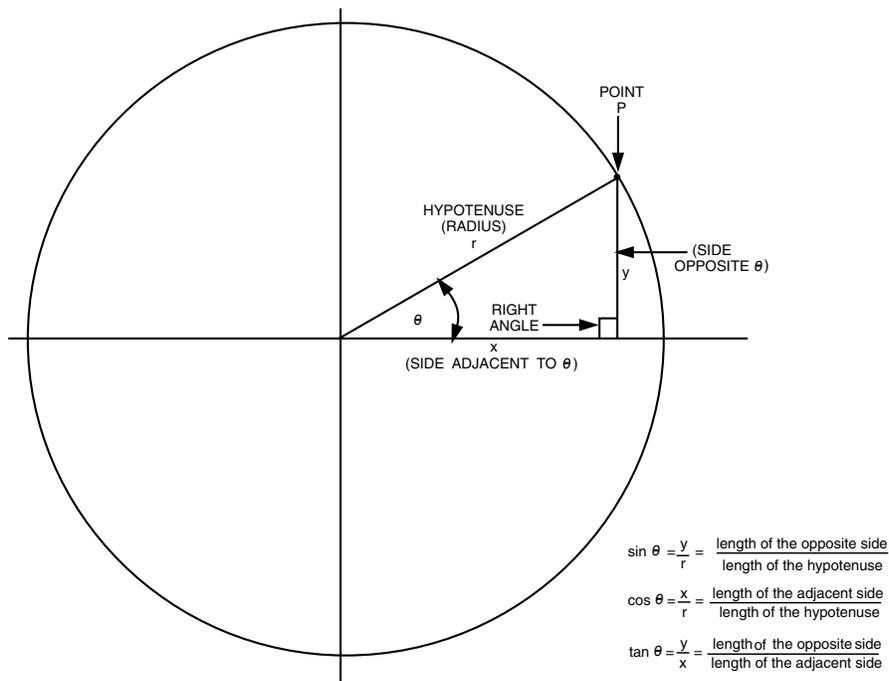
In trigonometry, angles are represented by the Greek letter theta ( $\theta$ ). The sine of an angle  $\theta$ , abbreviated as  $\sin \theta$ , is the ratio expressed when the side of a right triangle opposite the angle  $\theta$  is divided by the hypotenuse. Figure 12-1 shows these trigonometric relations.

Therefore, referring to figure 12-1:

**$\sin \theta = y/r$ , or the altitude (y) divided by the hypotenuse (r)**

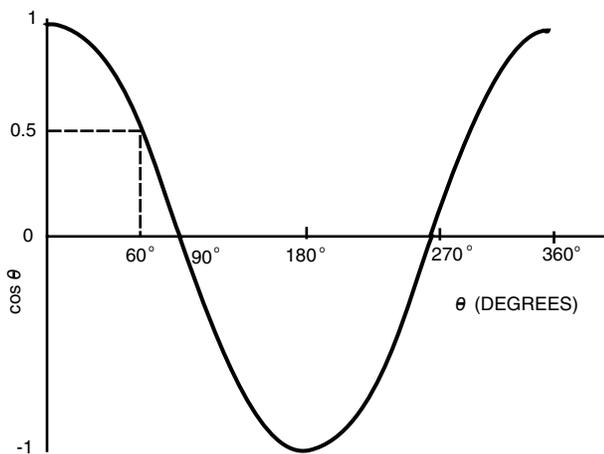
If the hypotenuse (r) is also the radius of a circle, point P moves along the circumference as the angle changes in size. As angle  $\theta$  increases, side y increases in length while the length of the hypotenuse (or radius) remains the same. Therefore, the value of the sine increases as the angle increases. Changes in the value of the sine corresponding to changes in the size of the angle are shown on the sine curve shown in figure 12-2. On the sine curve, the size of the angle is plotted horizontally and the value of the sine vertically.

At any angle, the vertical height between the baseline and the curve is the value of the sine of the angle. This curve shows that the value of the sine at  $30^\circ$  is half of the value of the sine at  $90^\circ$ . At  $0^\circ$ ,  $\sin \theta$  equals zero. At  $90^\circ$ ,  $\sin \theta$  equals one.



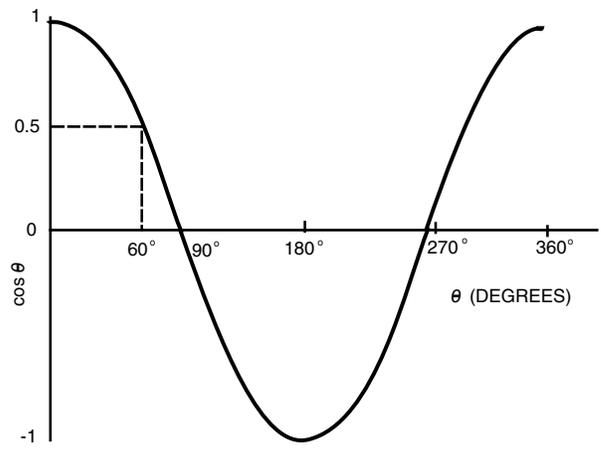
DCH1201

Figure 12-1. Trigonometric relationships.



DCH1203

Figure 12-2. Sine curve.



DCH1203

Figure 12-3. Cosine curve.

## Cosine

The cosine is the ratio expressed by dividing the side adjacent to the angle  $\theta$  by the hypotenuse. Therefore, referring to figure 12-1:

**$\cos \theta = x$  divided by  $r$  (the adjacent divided by the hypotenuse)**

In contrast to the sine, the cosine decreases as the angle  $\theta$  becomes larger. This relationship between the value of the cosine and the size of the angle is shown by the cosine curve shown in figure 12-3. At  $0^\circ$  the cosine equals one; at  $90^\circ$  the cosine equals zero; and at  $60^\circ$  the cosine is half the value of the cosine at  $0^\circ$ .

## Tangent

The tangent of the angle  $\theta$  is the ratio of the side opposite the angle  $\theta$  to the side adjacent. Again, referring to figure 12-1:

**$\tan \theta = y$  divided by  $x$  (the side opposite  $\theta$  divided by the side adjacent  $\theta$ )**

## PRINCIPLES OF PHYSICS

There are certain principles of physics that you need to know in order to have an adequate understanding of stability. You should be familiar with

such terms as *volume, density, weight, center of gravity, force, and moments.*

### Volume

The volume of any object is determined by the number of cubic feet or cubic units contained in the object. The underwater volume of a ship is found by determining the number of cubic feet in the part of the hull below the waterline.

### Density

The density of any material, solid or liquid, is obtained by weighing a unit volume of the material. For example, if you take 1 cubic foot of seawater and weigh it, the weight is 64 pounds or 1/35 of a ton (1 long ton equals 2,240 pounds). Since seawater has a density of 1/35 ton per cubic foot, 35 cubic feet of seawater weighs 1 long ton.

### Weight

If you know the volume of an object and the density of the material, the weight of the object is found by multiplying the volume by the density. The formula for this is as follows:

$$W = V \times D \text{ (weight = volume times density)}$$

When an object floats in a liquid, the weight of the volume of liquid displaced by the object is equal to the weight of the object. Thus, if you know the volume of the displaced liquid, the weight of the object is found by multiplying the volume by the density of the liquid.

*Example:*

If a ship displaces 35,000 cubic feet of salt water, the ship weighs 1,000 tons.

$$W = V \times D \text{ (weight = volume times density)}$$

$$W = 35,000 \text{ cubic feet} \times 1/35 \text{ ton per cubic foot}$$

$$W = 1,000 \text{ tons}$$

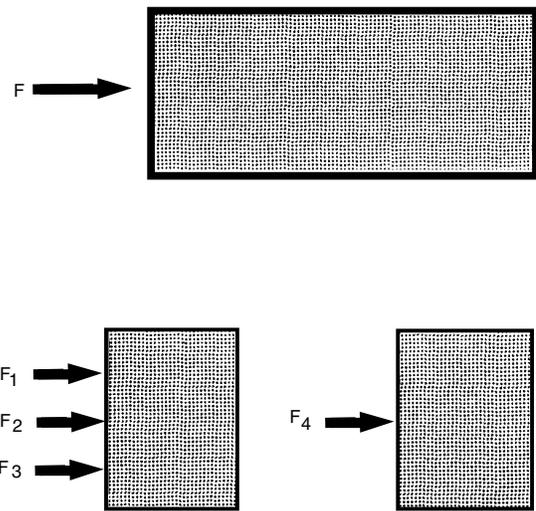
### Center of Gravity

The center of gravity (G) is the point at which all the weights of the unit or system are considered to be concentrated and have the same effect as that of all the component parts.

### Force

A force is a push or pull. It tends to produce motion or a change in motion. Force is what makes something start to move, speed up, slow down, or keep moving against resistance (such as friction). A force may act on an object without being in direct contact with it. The most common example of this is the pull of gravity. Forces are usually expressed in terms of *weight units*, such as *pounds, tons, or ounces.*

Figure 12-4 shows the action of a force on a body. An arrow pointing in the direction of the force is drawn to represent the force. The location and direction of the force being applied is known as the line of action. If a number of forces act together on a body, they may be considered as a single combined force acting in the same direction to produce the same overall effect. In this manner you can understand that  $F_4$  in figure 12-4 is the resultant or the sum of the individual forces  $F_1$ ,  $F_2$ , and  $F_3$ .



DCf1204

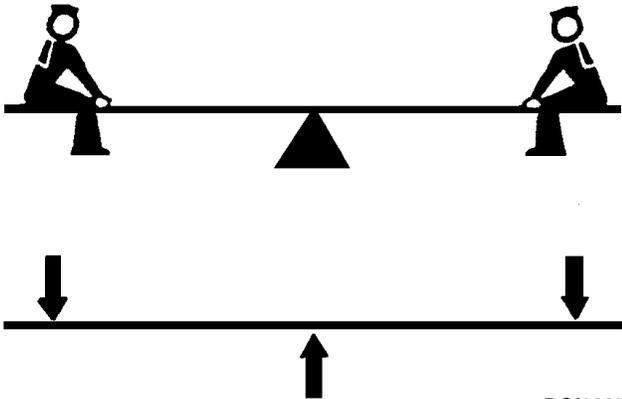
Figure 12-4. Lines indicating direction of force.

Whether you consider the individual forces  $F_1$ ,  $F_2$ , and  $F_3$ , or just  $F_4$  alone, the action of these forces on the object will move the body in the direction of the force.

To prevent motion or to keep the body at rest, you must apply an equal force in the same line of action but in the opposite direction to  $F_4$ . This new force and  $F_4$  will cancel each other and there will be no movement; the resultant force is zero. An example of this is a Sailor attempting to push a truck that is too heavy for him to move. Although the truck does not move, force is still being exerted.

## Moments

In addition to the size of a force and its direction of action, the location of the force is important. For example, if two persons of the same weight sit on opposite ends of a seesaw, equally distant from the support (fig. 12-5), the seesaw will balance. However, if one person moves, the seesaw will no longer remain balanced. The person farthest away from the support will move down because the effect of the force of his/her weight is greater.

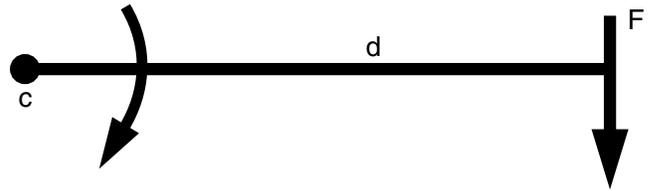


DCf1205

Figure 12-5. The balanced seesaw.

The effect of the location of a force is known as the **MOMENT OF FORCE**. It is equal to the force multiplied by the distance from an axis about which you want to find its effect. The moment of a force is the tendency of the force to produce rotation or to move the object around an axis. Since the force is expressed in terms of weight units, such as tons or pounds, and the moment is force times distance, the units for moment are expressed as foot-tons, foot-pounds, or inch-ounces.

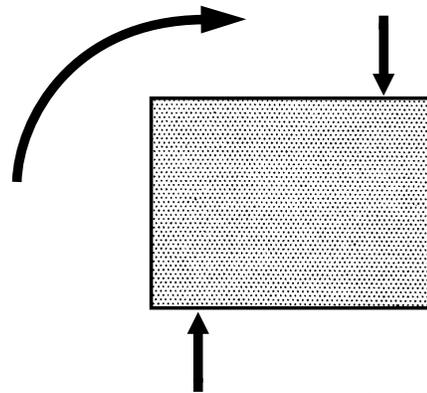
In figure 12-6 the moment of force (F) about the axis at point a is F times d; d being called the moment arm. The moment of a force can be measured about any point or axis; however, the moment differs according to the length of the moment arm. It should be noted that the moment of a force tends to produce rotary motion. In figure 12-6, for example, the force F produces a clockwise rotation. If, at the same time, an equal and opposite force produces a counterclockwise rotation, there will be no rotation; and the body is in equilibrium.



DCf1206

Figure 12-6. Diagram to illustrate the moment of force.

A special case of moments occurs when two equal and opposite forces not in the same line rotate a body. This system of two forces, as shown in figure 12-7, is termed a **COUPLE**. The moment of the couple is the product of one of the forces times the distance between them (fig. 12-8).



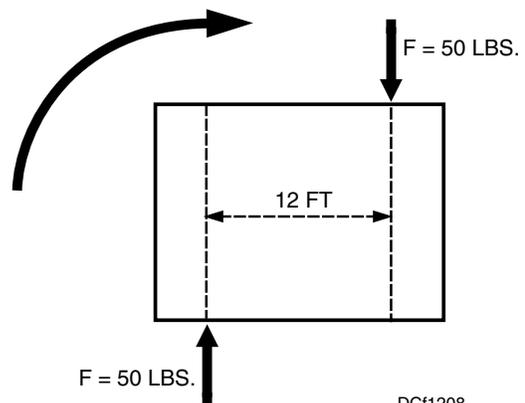
DCf1207

Figure 12-7. Equal and opposite forces acting on a body (not in the same line).

Calculation of the moment of the couple, as shown in figure 12-8, is as follows:

$$\text{The moment of the couple} = F \times d$$

Therefore, the moment of the couple is 50 feet times 12 pounds that equals 600 foot-pounds.

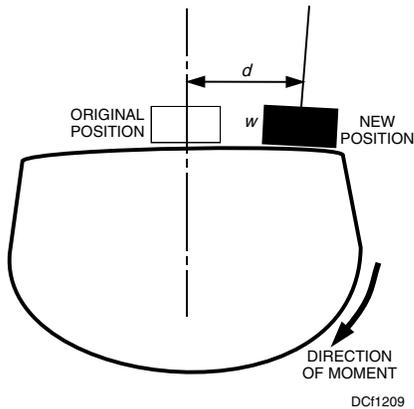


DCf1208

Figure 12-8. Diagram to show calculation of the moment of a couple.

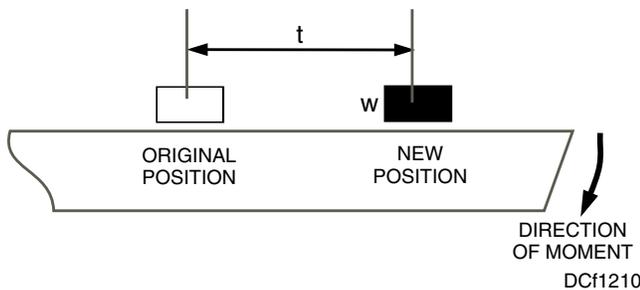
In one sense, a ship may be considered as a system of weights. If the ship is undamaged and floating in calm water, the weights are balanced and the ship is stable. However, the movement of weight on the ship causes a change in the location of the ship's center of gravity, and thereby affects the stability of the ship.

Figure 12-9 shows how an **INCLINING MOMENT** is produced when a weight is moved outboard from the centerline of the ship. If the object weighing 20 tons is moved 20 feet outboard from the centerline, the inclining moment will be equal to 400 foot-tons ( $F \times d$ , or  $20 \times 20$ ).



**Figure 12-9. Inclining moment produced by moving a weight outboard.**

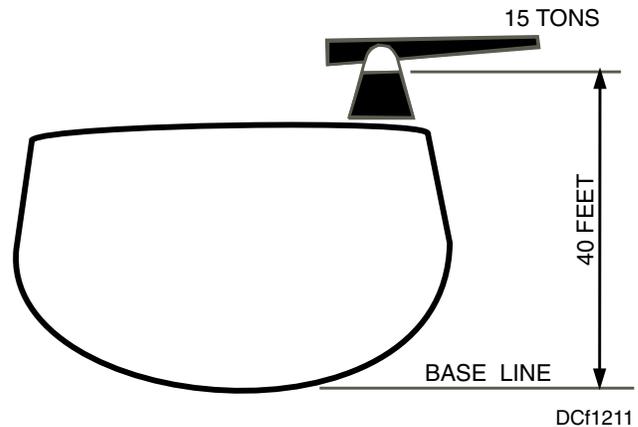
Figure 12-10 shows how a forward (or aft) movement of weight produces a **TRIMMING MOMENT**. Let's assume that a 20-ton weight is moved 50 feet forward; the trimming moment produced is  $20 \times 50$ , or 1,000 foot-tons.



**Figure 12-10. Trimming moment.**

It is also possible to calculate the **VERTICAL MOMENT** of any part of the ship's structure or of any weight carried on board. In calculating a vertical

moment, use the ship's baseline, or keel, as the axis. Figure 12-11 shows the calculation of the vertical moment of a 5-inch gun on the main deck of a ship. The gun weighs 15 tons and is located 40 feet above the keel. The vertical moment is thus  $15 \times 40$ , or 600 foot-tons.

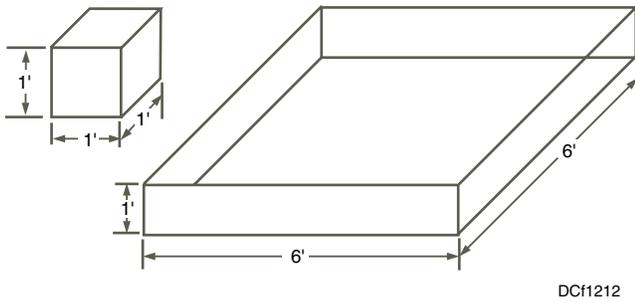


**Figure 12-11. Vertical moment.**

## BUOYANCY VERSUS GRAVITY

“Buoyancy” may be defined as the ability of an object to float. If an object of a given volume is placed under water and the weight of this object is **GREATER** than the weight of an equal volume of water, the object will sink. It sinks because the **FORCE** that buoys it up is less than the weight of the object. However, if the weight of the object is **LESS** than the weight of an equal volume of water, the object will rise. The object rises because the **FORCE** that buoys it up is greater than the weight of the object; it will continue to rise until it is partly above the surface of the water. In this position the object will float at such a depth that the submerged part of the object displaces a volume of water **EQUAL** to the weight of the object.

As an example, take the cube of steel shown in figure 12-12. It is solid and measures 1 foot by 1 foot by 1 foot. If you drop the steel cube into a body of water, the steel cube will sink because it weighs more than a cubic foot of water. But if you hammer this cube of steel into a flat plate 8 feet by 8 feet, bend the edges up 1 foot all-around, and make the corner seams watertight, this 6-foot by 6-foot by 1-foot box, as shown in figure 12-12, will float. In fact, it will not only float but will, in calm water, support an additional 1,800 pounds.



**Figure 12-12. A steel cube, and a box made from the same volume of steel.**

It is obvious, then, that the volume of the submerged part of a floating ship provides the buoyancy to keep the ship afloat. If the ship is at rest, the buoyancy (which is the weight of the displaced water) must be equal to the weight of the ship. For this reason, the weight of a ship is generally referred to as **DISPLACEMENT**, meaning the weight of the volume of water displaced by the hull.

Since weight (*W*) is equal to the displacement, it is possible to measure the volume of the underwater body (*V*) in cubic feet and multiply this volume by the weight of a cubic foot of seawater to determine what the ship weighs. This relationship may be written as the following:

$$(1) W = V \times \frac{1}{35}$$

$$(2) V = 35W$$

*V* = Volume of displaced seawater (in cubic feet)

*W* = Weight in tons

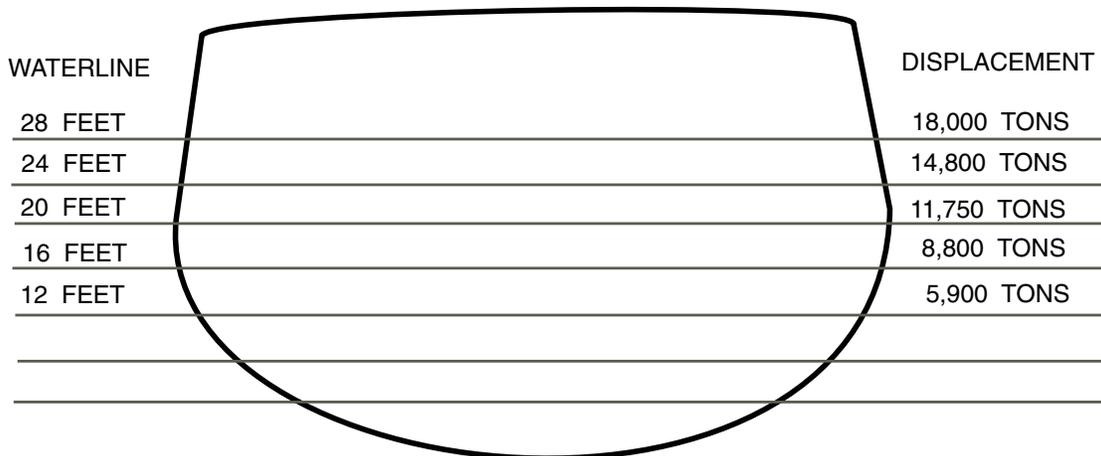
35 = Cubic feet of seawater per ton (For ships, the long ton of 2,240 pounds is used.)

It is obvious that displacement will vary with the depth of a ship's keel below the water line that is known as draft. As the draft increases, the displacement increases. This is indicated in figure 12-13 by a series of displacements shown for successive draft lines on the midship section of a ship. The volume of an underwater body for a given draft line can be measured in the drafting room by using graphic or mathematical means. This is done for a series of drafts throughout the probable range of displacements in which a ship is likely to operate. The values obtained are plotted on a grid on which feet of draft are measured vertically and tons of displacement horizontally. A smooth line is faired through the points plotted, providing a curve of displacement versus draft, or a **DISPLACEMENT CURVE** as it is generally called. An example of this for a typical warship is shown in figure 12-14.

To use the sample curve shown in figure 12-14 for finding the displacement when the draft is given, locate the value of the mean draft on the draft scale at the left. Then proceed horizontally across the diagram to the displacement curve. From this point proceed vertically downward and read the displacement from the scale. For example, if you have a mean draft of 26 feet, the displacement found from the curve is approximately 16,300 tons.

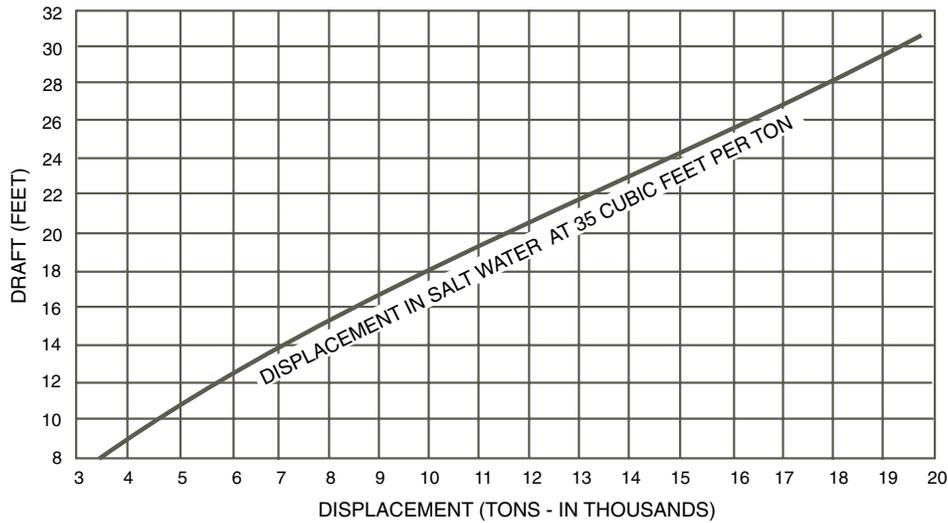
### Reserve Buoyancy

The volume of the watertight portion of the ship above the waterline is known as the ship's reserve buoyancy. Expressed as a percentage, reserve buoyancy is the ratio of the volume of the above-water body to the volume of the underwater body. Thus reserve buoyancy may be stated as a volume in cubic feet, as a ratio or percentage, or as an equivalent weight of seawater in tons. (In tons it is 1/35 of the volume in cubic feet of the above-water body.)



**Figure 12-13. Example of displacement data.**

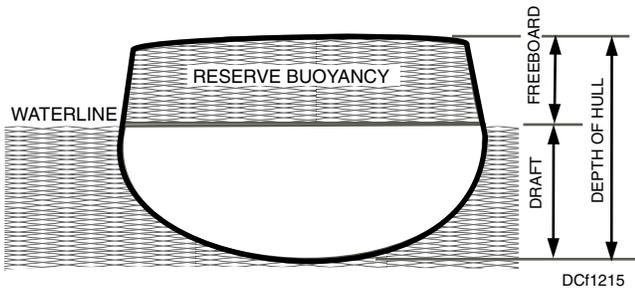
DCf1213



DCh1214

Figure 12-14. Example of a displacement curve.

Freeboard, a rough measure of reserve buoyancy, is the distance in feet from the waterline to the weather deck edge. Freeboard is calculated at the midship section. As indicated in figure 12-15, freeboard plus draft always equals the depth of the hull in feet.



DCh1215

Figure 12-15. Reserve buoyancy, freeboard, draft, and depth of hull.

When weight is added to a ship, draft and displacement increase in the same amount freeboard and reserve buoyancy decrease. It is essential to the seaworthiness of a ship to retain a substantial amount of reserve buoyancy. Some ships can take more than their own weight in flooding water aboard without sinking due to reserve buoyancy.

### Center of Buoyancy

When a ship is floating at rest in calm water, it is acted upon by two sets of forces: (1) the downward force of gravity and (2) the upward force of buoyancy.

The force of gravity is a resultant or composite force, including the weights of all portions of the ship's structure, equipment, cargo, and personnel. The force of gravity may be considered as a single force, which acts downward through the ship's center of gravity (G).

The force of buoyancy is also a composite force, which results from the pressure of the water on the ship's hull. A good example of this is immersing a container in a tank of water as shown in view A of figure 12-16. The container must be held under the water to keep it from rising. View B of figure 12-16 shows the position of the container when it is released.

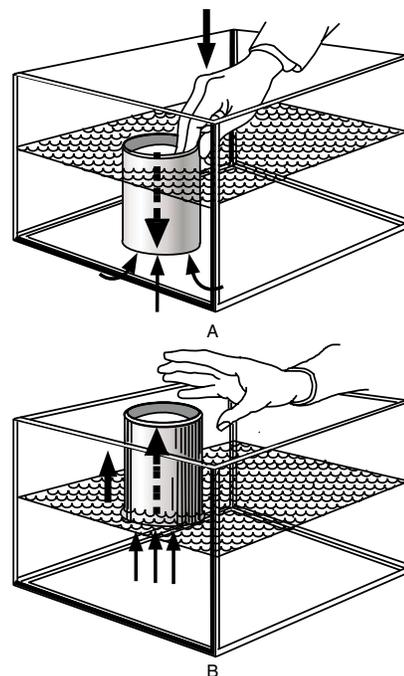
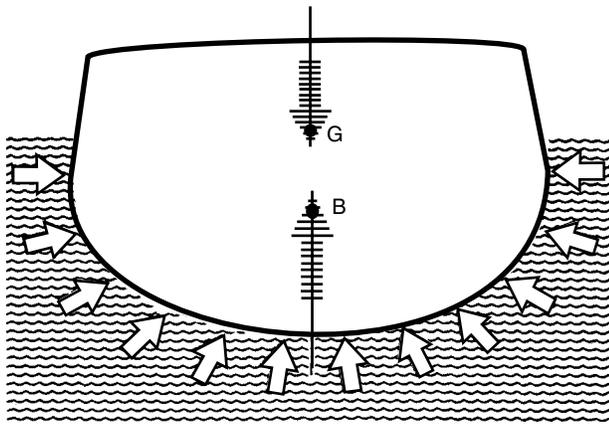


Figure 12-16. A. An immersed container; B. The container forced upward.

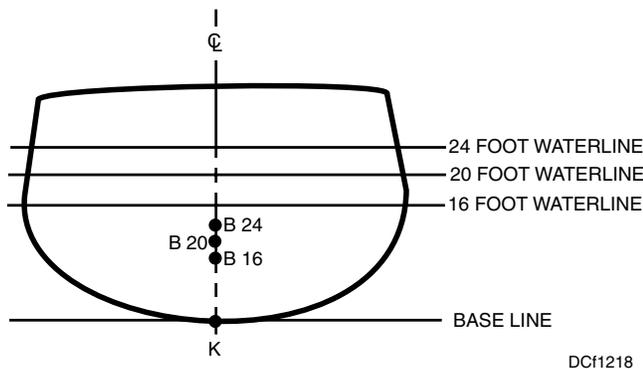
Horizontal pressures on the sides of a ship cancel each other under normal conditions, as they are equal forces acting in opposite directions (fig. 12-17). The vertical pressure may be regarded as a single force—the force of buoyancy acting vertically upward through the CENTER OF BUOYANCY (B).



DC11217

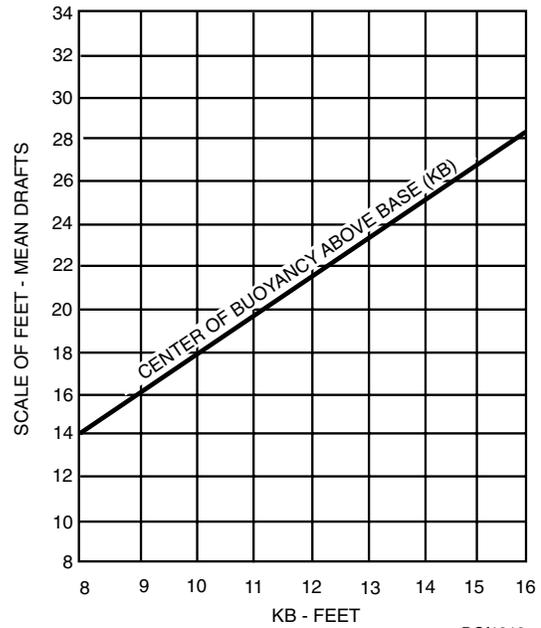
Figure 12-17. Relationship of the forces of buoyancy and gravity.

When a ship is at rest in calm water, the forces of buoyancy (B) and gravity (G) are equal and lie in the same vertical line, as shown in figure 12-17. The center of buoyancy, being the geometric center of the ship's underwater body, lies on the centerline and usually near the midship section, and its vertical height is usually a little more than half the draft. As the draft INCREASES, B rises with respect to the keel. Figure 12-18 shows how different drafts will create different values of the HEIGHT OF THE CENTER OF BUOYANCY FROM THE KEEL (KB). A series of values for KB (the center of buoyancy from the keel) is obtained and these values are plotted on a curve to show KB versus draft. Figure 12-19 shows an example of a KB curve for a warship.



DC11218

Figure 12-18. Successive centers of buoyancy (B) for different drafts.



DC11219

Figure 12-19. Curve of center of buoyancy above base.

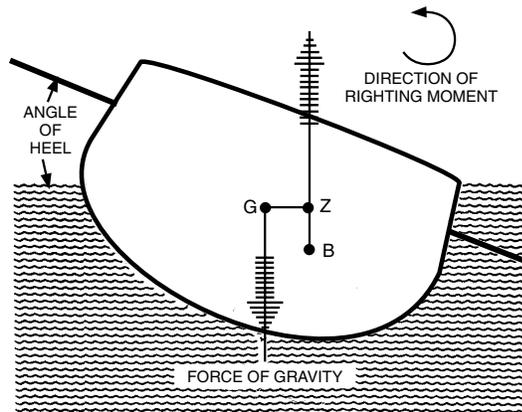
To read KB when the draft is known, start at the proper value of the draft on the scale at the left (fig. 12-19) and proceed horizontally to the curve. Then drop vertically downward to the baseline (KB). Thus, if our ship were floating at a mean draft of 19 feet, the KB found from the chart would be approximately 11 feet.

### Inclining Moments

A ship may be disturbed from rest by conditions which tend to make it heel over to an angle. These conditions include such things as wave action, wind pressures, turning forces when the rudder is put over, recoil of gunfire, impact of a collision or enemy hit, shifting of weights on board, and addition of off-center weights. These conditions exert heeling moments on the ship that may be temporary or continuous.

When a disturbing force exerts an inclining moment on a ship, there is a change in the shape of the ship's underwater body. The underwater volume is relocated, its bulk being shifted in the direction of the heel. This condition causes the center of buoyancy (B) to leave the ship's centerline and shift in the direction of the heel. (The center of buoyancy moves to the geometric center of the new underwater body.) As a result, the lines of action of the forces of buoyancy and gravity separate and in doing so exert a MOMENT on the ship. This moment tends to restore the ship to an even keel.

If you study figure 12-20, you will notice that a RIGHTING or RESTORING MOMENT is present. This righting moment is caused by the two equal and opposite forces, each of W tons (displacement) magnitude, separated by a distance GZ, which constitutes the LEVER ARM OF MOMENT. Figure 12-20 shows that the ship is stable because the center of buoyancy (B) has shifted far enough to position the buoyant force where it tends to restore the ship to an even keel or an upright position.



DC11220

Figure 12-20. Development of righting moment when a stable ship inclines.

A moment is the product of a force tending to produce a rotation about an axis times its distance from the axis. If two equal and opposite forces are separated by a distance, the moment will become a couple which is measured by ONE of the forces times the distance that separates them. The RIGHTING MOMENT of a ship is therefore the product of the force of buoyancy times the distance GZ (fig. 12-20) that separates the forces of buoyancy and gravity. It may also be expressed as the force of gravity (weight of the ship) times GZ. The distance GZ is known as a ship's RIGHTING ARM. Putting this into mathematical terms, you have the following:

Righting moment =  $W \times GZ$  (expressed in foot-tons)

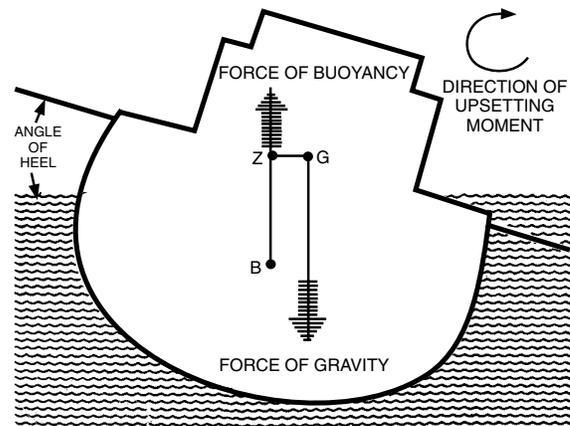
Where:

$W$  = displacement in tons

$GZ$  = righting arm in feet

For example, if a ship displaces 10,000 tons and has a 2-foot righting arm at 40° inclination, the righting moment is 10,000 tons times 2 feet, or 20,000 foot-tons. These 20,000 foot-tons represent the force, which tends to return the ship to an upright position.

However, it is possible for conditions to exist which do not permit B to move far enough in the direction in which the ship rolls to place the buoyant force outboard of the force of gravity. The moment produced will tend to upset the ship, rendering it unstable. Figure 12-21 shows an unstable ship in which the relative positions of B and G produce an UPSETTING MOMENT. In this illustration it is obvious that the cause of the upsetting moment is the high position of G (center of gravity) and the GEOMETRIC CENTER OF THE UNDERWATER BODY (B—the center of buoyancy).

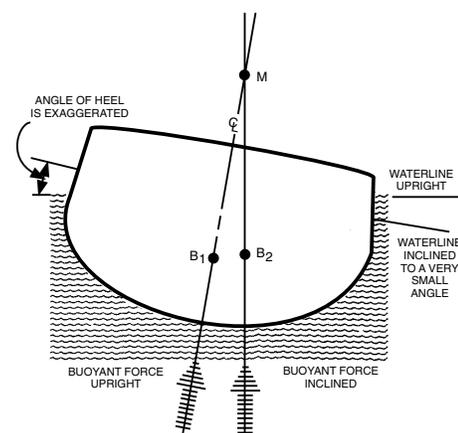


DC11221

Figure 12-21. Development of an upsetting moment when an unstable ship inclines.

### Metacenter

A ship's METACENTER (M) is the intersection of two successive lines of action of the force of buoyancy, as the ship heels through a very small angle. Figure 12-22 shows two lines of buoyant force. One of these represents the ship on even keel, the other at a small angle of heel. The point where they intersect is the initial position of the metacenter. When the angle of heel is greater than the angle used to compute the metacenter, M moves off the centerline and the path of movement is a curve.



DC11222

Figure 12-22. The metacenter.

The INITIAL position of the metacenter is most useful in the study of stability, because it provides a reference point when the ship is upright and most stable. In our discussion we will refer to initial position of M. The distance from the center of buoyancy (B) to the metacenter (M) when the ship is on even keel is the METACENTRIC RADIUS.

### Metacentric Height

The distance from the center of gravity (G) to the metacenter is known as the ship's METACENTRIC HEIGHT (GM). Figure 12-23, view A, shows a ship heeled through a small angle (the angle is exaggerated in the drawing), establishing a metacenter at M. The ship's righting arm is GZ, which is one side of the triangle GZM. In this triangle GZM, the angle of heel is at M. The side GM is perpendicular to the waterline at even keel, and ZM is perpendicular to the waterline when the ship is inclined.

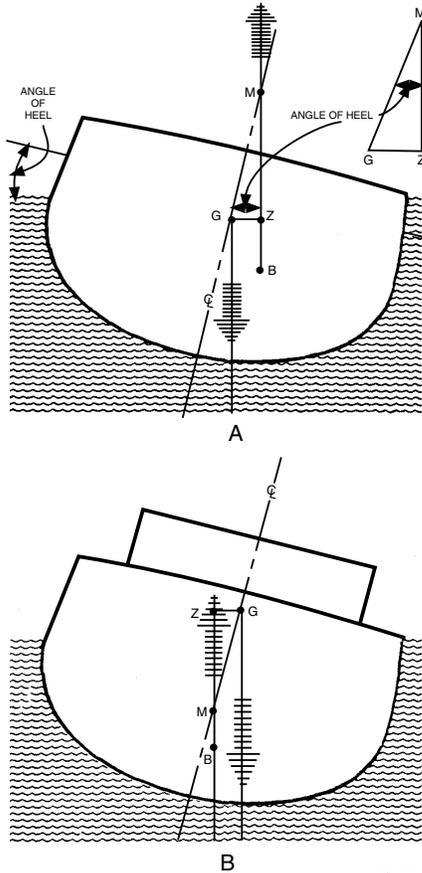


Figure 12-23. A. Stable condition, G is below M; B. Unstable condition, G is above M.

It is evident that for any given angle of heel, there will be a definite relationship between GM and GZ because  $GZ = GM \sin \theta$ . Thus GM acts as a measure of GZ, the righting arm.

The ship's METACENTRIC HEIGHT (GM) is not only a measure of the ship's RIGHTING ARM (GZ) but is also an indication of whether the ship is stable or unstable. If M is above G, the metacentric height is positive, the moments which develop when the ship is inclined are RIGHTING MOMENTS, and the ship is stable, as shown in view A of figure 12-23. But if M is below G, the metacentric height is negative, the moments that develop are UPSETTING MOMENTS, and the ship is unstable, as shown in view B of figure 12-23.

### Influence of Metacentric Height

If the metacentric height (GM) of a ship is large, the righting arms that develop, at small angles of heel, will be large. Such a ship is "stiff" and will resist roll. However, if the metacentric height of a ship is small, the righting arms that develop will be small. Such a ship is tender and will roll slowly.

In ships, large GM and large righting arms are desirable for resistance to damage. However, a smaller GM is sometimes desirable for a slow, easy roll that allows for more accurate gunfire; therefore, the GM value for a naval ship is the result of compromise.

### Inclining Experiment

The ship designer uses calculations to determine the vertical position of the center of gravity. From available plans and data, the various items that go to make up the ship and its load are tabulated. The ship can be considered as consisting of the various parts of the structure, machinery, and equipment. The load is comprised of fuel, oil, water, ammunition, and sundry stores aboard.

Although the position of the center of gravity as estimated by calculation is sufficient for design purposes, an accurate determination is required to establish the overall stability of the ship when it is operating. Therefore, an inclining experiment is performed to obtain accurately the vertical height of the center of gravity above the keel (KG) when the ship is completed. An inclining experiment consists of moving one or more large weights across the ship and measuring the angle of list produced. This angle of list usually does not exceed  $2^\circ$ . The ship should be in the best possible condition for the inclining. The naval shipyard or building yard at which the inclining experiment is to be performed issues a memorandum to

the commanding officer of the ship outlining the necessary work to be done by the ship's force and by the yard to prepare the ship for the inclining.

The results of the experiment are furnished to each ship as a "booklet of inclining experiment data." This booklet contains data on displacement, the center of gravity above the keel (KG), and overall stability for the operating conditions of load. Detailed information on the inclining experiment can be obtained from *Naval Ships' Technical Manual (NSTM)*, chapter 096, "Weights and Stability."

---

### REVIEW QUESTIONS

- Q1. Detailed information on the laws of mathematics and physics used to determine the buoyancy and stability of a ship are provided in *Naval Ships' Technical Manual (NSTM)*, chapter 079, volume 1, and in *NSTM*, chapter 096.
1. True
  2. False
- Q2. Which of the following trigonometric functions is NOT used for making calculations to determine a ship's stability?
1. Cosine
  2. Sine
  3. Tangent
  4. Cotangent
- Q3. Which of the following terms best defines force multiplied by the distance from an axis about which you want to find its effect?
1. *Moment of force*
  2. *Friction*
  3. *Ballast*
  4. *Inclining moment*
- Q4. The volume of water that is moved by the hull of a ship is known as "displacement."
1. True
  2. False

- Q5. What measurement is known by the term *freeboard*?
1. Distance in feet from the keel to the waterline
  2. Distance from the waterline to the weather deck edge
  3. Distance from the bow to the stern
  4. Distance from the portside to the starboard side of the ship
- Q6. Which of the following information is NOT contained in the "booklet of inclining experiment data?"
1. Data on displacement
  2. The center of gravity above the keel
  3. Reserve buoyancy
  4. Overall stability

---

### ANALYSIS OF STABILITY

**Learning Objectives:** Recall the laws of physics and trigonometry used to determine stability and buoyancy of a ship; and the effects of buoyancy, gravity, and weight shifts on ship stability.

To analyze stability principles, you must be familiar with the terms, definitions, and equations that are used to express important relationships. These are listed below.

- G, the ship's center of gravity, is the point at which all weights of the ship may be considered to be concentrated. The force of gravity is considered as acting straight downward, through the center of gravity, at right angles to the waterline.
- B, the ship's center of buoyancy, is at the geometric center of the ship's underwater hull. When a ship is at rest in calm water, the forces of B and G are equal and opposite, and the points B and G lie in the same vertical line. When the ship is inclined, B and G move apart, since B moves off the ship's centerline as a result of the change in the shape of the underwater hull.
- M, the ship's metacenter, is a point established by the intersection of two successive lines of buoyant force as the ship heels through a very small angle.

- GM, metacentric height, is the distance from G to M; it is measured in feet. Z is the point at which a line, through G, parallel to the waterline, intersects the vertical line through B.
- GZ, the distance from G to Z, is the ship's righting arm; it is measured in feet. For small angles of heel, GZ may be expressed by the equation
- $GZ = GM \sin \theta$
- W is the weight (displacement) of the ship; it is measured in long tons.
- K is a point at the bottom of the keel, at the midship section, from which all vertical measurements are made.
- KB is the vertical distance from K to the center of buoyancy when the ship is upright. KB is measured in feet.
- KG is the vertical distance from K to the ship's center of gravity when the ship is upright. KG is measured in feet.
- KM is the vertical distance from K to the metacenter when the ship is upright. KM is measured in feet.

The RIGHTING MOMENT of a ship is W times GZ, that is, the displacement times the righting arm.

Righting moments are measured in foot-tons. Since the righting arm (GZ) is equal to GM times  $\sin \theta$ , for small values of  $\theta$ , you can say that the righting moment is equal to W times GM times  $\sin \theta$ . Because of the relationship between righting arms and righting moments, it is obvious that stability may be expressed either in terms of GZ or in righting moments. However, you must be very careful not to confuse righting arms with righting moments; they are NOT identical.

### STABILITY CURVES

When a series of values for GZ (the ship's righting arm) at successive angles of heel are plotted on a graph, the result is a STABILITY CURVE. The stability curve, as shown in figure 12-24, is called the CURVE OF STATIC STABILITY. The word *static* indicates that it is not necessary for the ship to be in motion for the curve to apply. If the ship is momentarily stopped at any angle during its roll, the value of GZ given by the curve will still apply.

#### NOTE

The stability curve is calculated graphically by design engineers for values indicated by angles of heel above 7°.

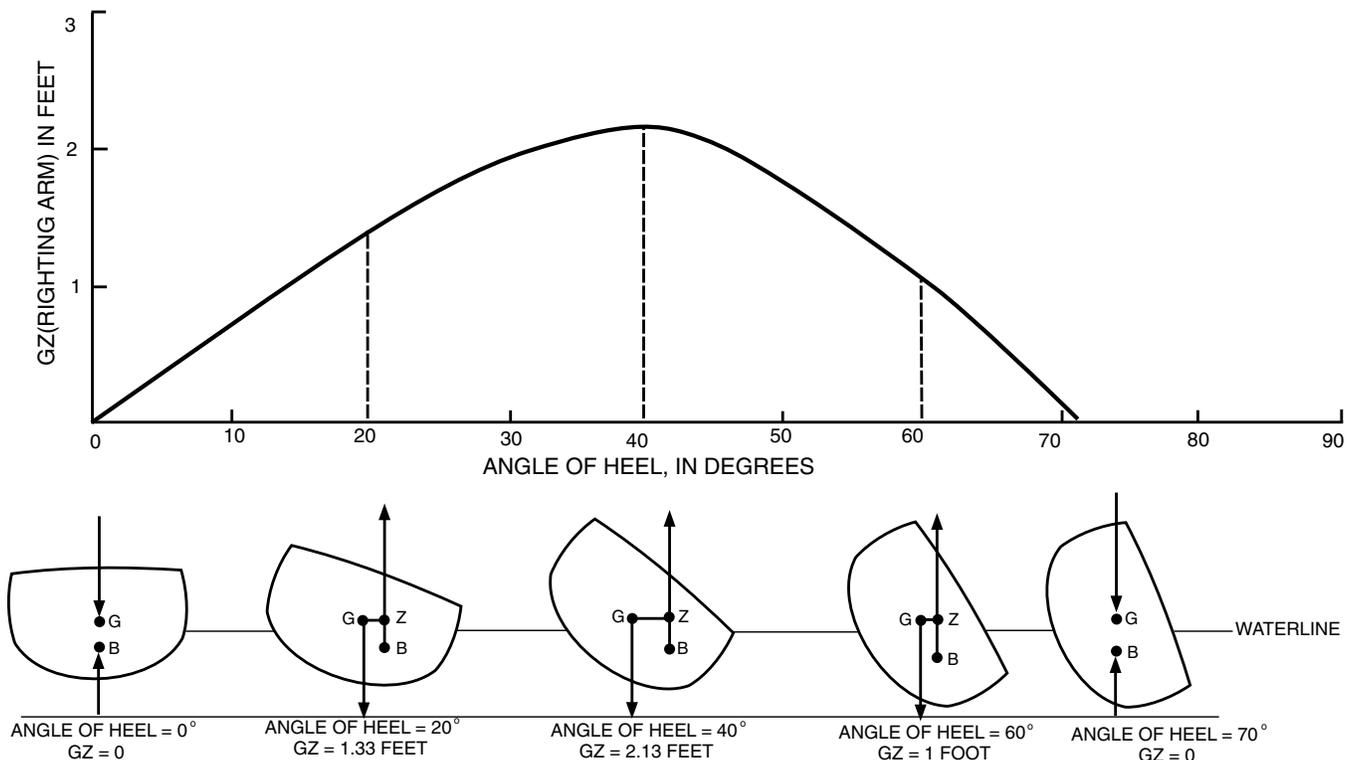


Figure 12-24. Curve of static stability.

DC11224

To understand this stability curve, it is necessary to consider the following facts:

1. The ship's center of gravity does NOT change position as the angle of heel is changed.
2. The ship's center of buoyancy is always at the geometric center of the ship's underwater hull.
3. The shape of the ship's underwater hull changes as the angle of heel changes.

If these three facts are considered collectively, you will see that the position of G remains constant as the ship heels through various angles, but the position of B changes according to the angle of inclination. When the position of B has changed so that B and G are not in the same vertical line, a righting arm GZ must exist. The length of this righting arm depends upon the angle at which the ship is inclined (fig. 12-25). GZ increases as the angle of heel increases, up to a certain point. At about an angle of 40°, the rate of increase of GZ begins to level off. The value of GZ diminishes and finally reaches zero at a very large angle of heel.

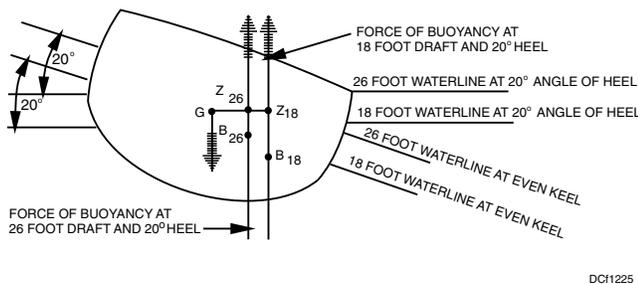


Figure 12-25. Effect of draft on righting arm.

### Effect of Draft on Righting Arm

A change in displacement will result in a change of draft and freeboard; and B will shift to the geometric center of the new underwater body. At any angle of inclination, a change in draft causes B to shift both horizontally and vertically with respect to the keel. The horizontal shift in B changes the distance between B and G, and thereby changes the length of the righting arm, GZ. Thus, when draft is increased, the righting arms are reduced throughout the entire range of stability. Figure 12-25 shows how the righting arm is reduced when the draft is increased from 18 feet to 26 feet when the ship is inclined at an angle of 20°.

A reduction in the size of the righting arm usually means a decrease in stability. When the reduction in GZ is caused by increased displacement, however, the total effect on stability is more difficult to evaluate. Since the RIGHTING MOMENT is equal to W times GZ, it will be increased by the gain in W at the same time that it is decreased by the reduction in GZ. The gain in the righting moment, caused by the gain in W, does not necessarily compensate for the reduction in GZ.

In summary, there are several ways in which an increase in displacement affects the stability of a ship. Although these effects occur at the same time, it is best to consider them separately. The effects of increased displacement are the following:

1. RIGHTING ARMS (GZ) are decreased as a result of increased draft.
2. RIGHTING MOMENTS (foot-tons) are decreased as a result of decreased GZ.
3. RIGHTING MOMENTS are increased as a result of the increased displacement (W).

### Cross Curves of Stability

The position of the center of buoyancy at any given angle of inclination depends upon the draft. As the draft increases, the center of buoyancy moves closer to the center of gravity, thereby reducing the length of the righting arms. To determine this effect, the design activity inclines a line drawing of the ship's lines at a given angle, and then lays off a series of waterlines on it. These waterlines are chosen at evenly spaced drafts throughout the probable range of displacements. For each waterline the value of the righting arm is calculated, using an ASSUMED center of gravity, rather than the TRUE center of gravity. A series of such calculations is made for various angles of heel—usually 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90°—and the results are plotted on a grid to form a series of curves known as the CROSS CURVES OF STABILITY.

Figure 12-26 is an example of a set of cross curves. Note that, as draft and displacement increase, the curves all slope downward, indicating increasingly smaller righting arms.

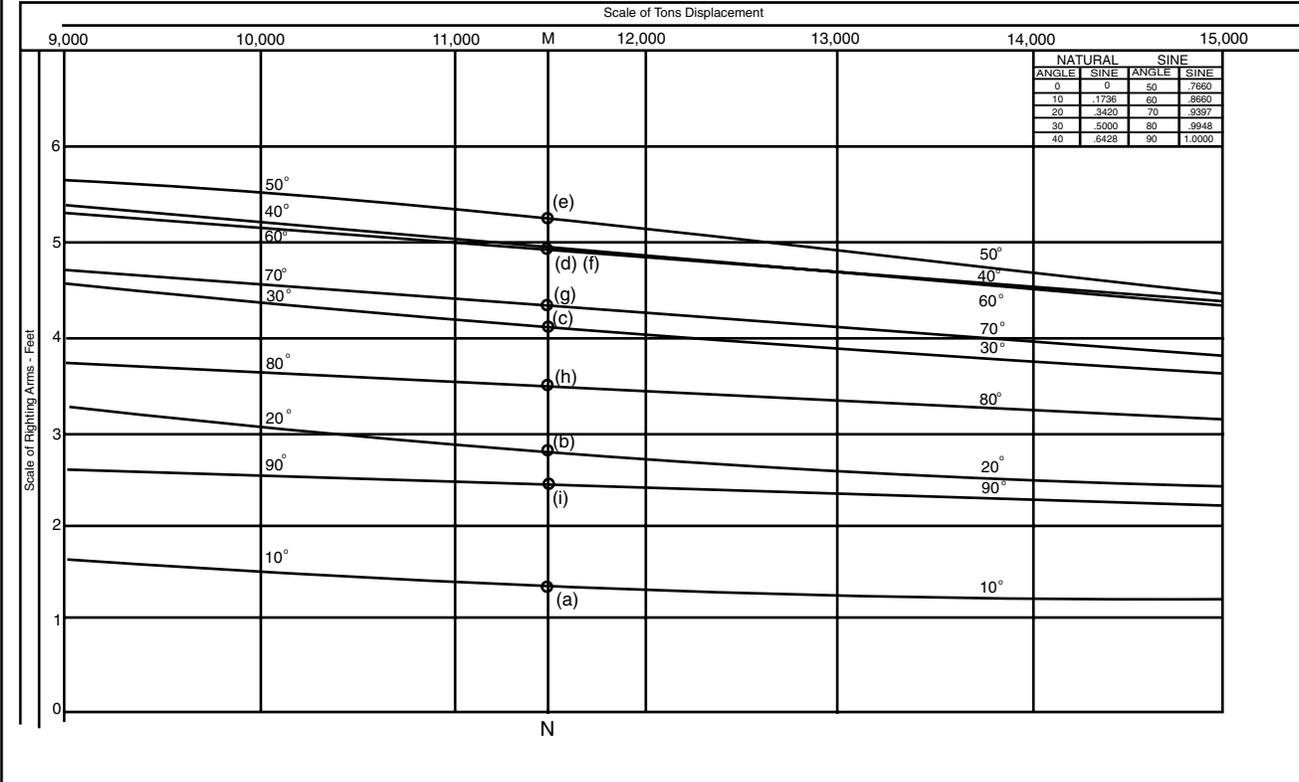
The cross curves are used in the preparation of stability curves. To take a stability curve from the cross curves, draw a vertical line (such as line MN in fig. 12-26) on the cross curve sheet at the displacement that corresponds to the mean draft of the ship. At the intersection of this vertical line with each cross curve,

Axis assumed ..... feet above base line

Vessel considered water-tight to .....

Taken from Plan No. ....

Buships No. ....



DCf1226

Figure 12-26. Example of cross curves of stability.

read the corresponding value of the righting arm on the vertical scale at the left. Then plot this value of the righting arm at the corresponding angle of heel on the grid for the stability curve. When you have plotted a series of such values of the righting arms from 10° to 90° of heel, draw a smooth line through them and you have the UNCORRECTED stability curve for the ship at that particular displacement.

In figure 12-27, curve A represents an uncorrected stability curve for the ship while operating at 11,500 tons displacement, taken from the cross curves shown in figure 12-26. This stability curve cannot be used in its present form, since the cross curves are made up on the basis of an assumed center of gravity. In actual operation, the ship's condition of loading will affect its displacement and therefore the location of the

ship's center of gravity (G). To use a curve taken from the cross curves, therefore, it is necessary to correct the curve for the ACTUAL height of G above the keel (KG). If the distance KG is not known and a number of weights have been added to or removed from a ship, KG can be found by the use of vertical moments. A vertical moment is the product of the weight times its vertical height above the keel. As far as the new center of gravity is concerned, when a weight is added to a system of weights, the center of gravity can be found by taking moments of the old system plus that of the new weight and dividing this total moment by the total final weight. Detailed information concerning changes in the center of gravity of a ship can be obtained from *Naval Ships' Technical Manual (NSTM)*, chapter 096.

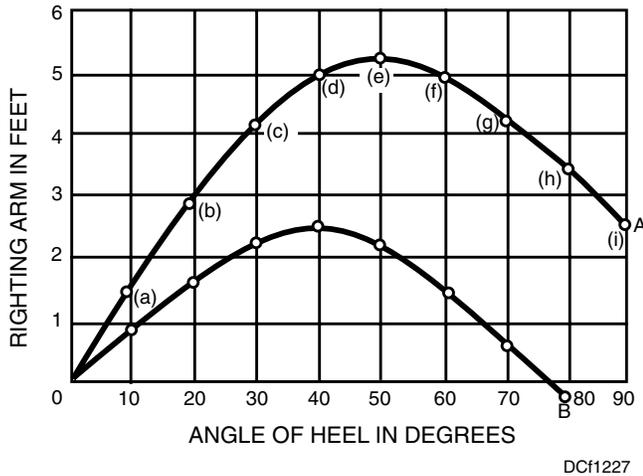


Figure 12-27. A. Uncorrected stability curve taken from cross curves; B. Corrected stability curve.

Suppose that the cross curves are made up on the basis of an assumed KG of 20 feet and that you determine that the actual KG is 24 feet for the particular condition of loading. This means that the true G is 4 feet higher than the assumed G and that the righting arm (GZ) at each angle of inclination will be SMALLER than the righting arm shown in figure 12-27 (curve A) for the same angle. To find the new value of GZ for each angle of inclination, multiply the increase in KG (4 feet) by the sine of the angle of inclination, and SUBTRACT this product from the value of GZ shown on the cross curves or on the uncorrected stability curve. In order to facilitate the correction of the stability curves, a table showing the necessary sines of the angles of inclination is included on the cross curves form.

Next, find the corrected values of GZ for the various angles of heel shown on the stability curve (A) in figure 12-27, and plot them on the same grid to make the corrected stability curve (B) shown in figure 12-27.

At 10°, the uncorrected value of GZ is 1.4; therefore, the corrected GZ at 10° is 1.4 minus (4 x 0.1736), or 0.7056.

At 20°, the uncorrected value of GZ is 2.8; therefore, the corrected GZ at 20° is 2.8 minus (4 x 0.3420), or 1.4320.

Repeating this process at 30°, 40°, 50°, 60°, 70°, and 80°, the following values are obtained:

- At 30°, the corrected GZ = 2.2000
- At 40°, the corrected GZ = 2.3288
- At 50°, the corrected GZ = 2.2360
- At 60°, the corrected GZ = 1.4360

At 70°, the corrected GZ = 0.5412

At 80°, the corrected GZ = minus 0.4392

It is not necessary to figure the corrected GZ at 90°, since the value is already negative at 80°. When the values from 10° through 80° are plotted on the grid and joined with a smooth curve, the CORRECTED stability curve (B) shown in figure 12-27 results. As you can see, the corrected curve shows maximum stability to be at 40°; it also shows that an upsetting arm, rather than a righting arm, generally exists at angles of heel in excess of 75°.

### EFFECTS OF LOOSE WATER

When a tank or a compartment in a ship is partially full of liquid that is free to move as the ship heels, the surface of the liquid tends to remain level. The surface of the free liquid is referred to as FREE SURFACE. The tendency of the liquid to remain level as the ship heels is referred to as FREE SURFACE EFFECT. The term LOOSE WATER is used to describe liquid that has a free surface; it is NOT used to describe water or other liquid that completely fills a tank or compartment and thus has no free surface.

#### Free Surface Effect

Free surface in a ship causes a reduction in GM, due to a change in the center of gravity, and a consequent reduction in stability. The free surface effect is separate from and independent of any effect that may result merely from the addition of the weight of the liquid. When free surface exists, a free surface correction must be included in stability calculations. However, when a tank is completely filled so that there is no free surface, the liquid in the tank may be treated as a solid; that is, the only effect of the liquid on stability is the effect of its weight at its particular location.

To understand the actions that occur because of free surface effect, use a centerline compartment that is partially full of water, as shown in figure 12-28, as an example.

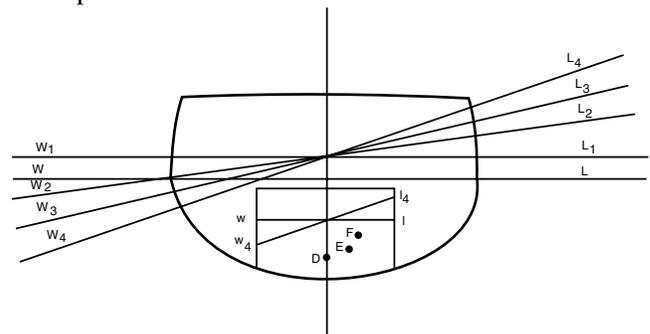


Figure 12-28. Effects of free surface.

To begin with, the ship is floating on an even keel at waterline  $WL$ . Then the compartment is flooded to waterline  $W_1L_1$ . Assuming that the water enters the compartment instantaneously and that it is instantaneously frozen solid, the effects of this frozen body of water are the same as if a solid weight had been added. The ship undergoes parallel sinkage and comes to rest at a new waterline  $W_1L_1$ .

Now suppose that an outside force acts on the ship, causing it to heel over at a small angle of list to a new waterline  $W_2L_2$ . If at the same time the liquid is freed from its frozen state, it will run toward the low side of the compartment until the surface of the water in the compartment is parallel to the existing waterline  $W_2L_2$ . A wedge of liquid is thus shifted from one side of the compartment to the other; as a result, the center of gravity of the liquid is shifted from  $D$  to  $E$ . As the center of gravity of the liquid is shifted outboard, an additional inclining moment is created. This causes the ship to list to a new waterline  $W_3L_3$ .

The additional list, in turn, causes a further shift of the liquid in the compartment and a further shift of the center of gravity of the liquid. As the center of gravity of the liquid shifts to  $F$ , another inclining moment is created and the ship lists even more. Eventually the ship will come to rest with a waterline such as  $W_4L_4$ . This will occur when the righting moment of the ship is equal to the combined effects of (1) the original inclining moment created by the outside force and (2) the inclining moment created by the shift of liquid within the compartment.

### Location of Free Surface

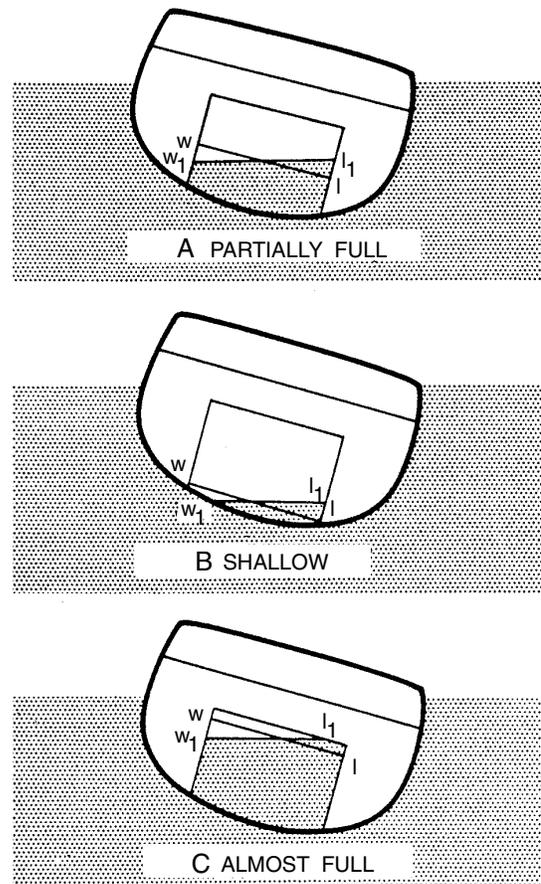
The free surface effect is independent of the location of the free surface within the ship. A free surface with a certain length and breadth will, at any given angle of heel, cause the same reduction in  $GM$  (and, therefore, the same loss of stability) no matter where it is in the ship—forward or aft, high or low, on the centerline or off the centerline.

### Depth of Loose Water

The free surface effect of a given area of loose water at a given angle of heel does NOT depend upon the depth of the loose water in the tank or compartment, unless the loose water is shallow enough or deep enough to cause the effect known as “pocketing” of the free surface. Pocketing occurs when the free surface of the liquid comes in contact with the deck or the

overhead and causes a reduction in the breadth of the free surface.

To understand how pocketing of the free surface reduces the free surface effect, study figure 12-29. View A of figure 12-29 shows a compartment in which the free surface effect is NOT influenced by the depth of the loose water. The compartment shown in view B, however, contains only a small amount of water. When the ship heels sufficiently to reduce the waterline in the compartment from  $w_1l_1$  to  $W_1L_1$ , the breadth of the free surface is reduced and the free surface effect is thereby reduced. A similar reduction in free surface effect occurs in the almost full compartment shown in view C, again because of the reduction in the breadth of the free surface. As figure 12-29 shows, the beneficial effect of pocketing is greater at larger angles of heel.



DC11229

Figure 12-29. Diagram to illustrate pocketing of free surface.

The reduction in free surface effect that results from pocketing is NOT taken into consideration when evaluating stability. Since pocketing improves stability, neglecting this factor in stability calculations provides a margin of safety; however, in centerline deep tanks on

some ships in which the tank is higher than wide, the opposite may be true. The normal practice of maintaining the fuel oil tanks 95 percent full takes advantage of the fact that pocketing occurs, at very small angles of heel, when a compartment is almost full.

### Length and Breadth of Free Surface

The athwartship breadth of a compartment has a great influence on the reduction in GM caused by the free surface effect. This influence is shown by the following formula:

$$\text{Rise in } G = \frac{b^3 l}{12(35W)}$$

Where b = athwartship breadth of compartment

l = fore-and-aft length of compartment

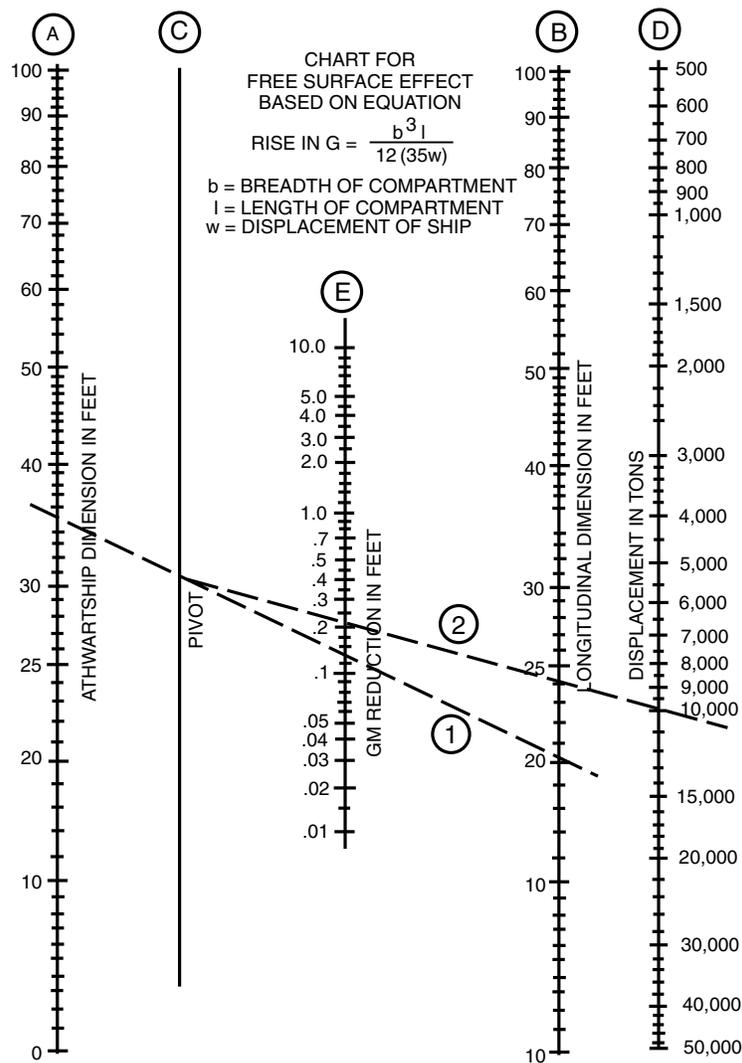
W = displacement of ship

As indicated by this formula, the free surface effect varies as the cube of the breadth (b) but only as the first power of the length (l). Because of this relationship, a single bulkhead that cuts a compartment in half in a fore-and-aft direction will quarter the free surface effect.

### Chart for Calculating Free Surface Effect

To avoid having to make calculations from the formula given in the previous section, a free surface effect chart based on this formula is used to find the reduction in GM that occurs as a result of free surface. Such a chart is shown in figure 12-30.

To use this chart, draw a straight line from the appropriate point on the ATHWARTSHIP DIMENSION scale (A) to the appropriate point on the



DCf1232

Figure 12-30. Chart for calculating free surface effect.

LONGITUDINAL DIMENSION scale (B); this line will intersect the pivot scale. Then draw a second straight line from the point of intersection on the pivot scale (C) to the appropriate point on the displacement scale (D). The point at which this second straight line intersects the GM reduction scale (E) gives you the reduction in GM (in feet) that is caused by the free surface.

For example, assume that you want to find what reduction in GM is caused by free surface effect in a partially flooded compartment that is 35 feet athwartships and 20 feet fore-and-aft, in a ship of 10,000 tons displacement. Draw the first straight line from the 35-foot point on the athwartship dimension scale to the 20-foot point on the longitudinal dimension scale. Then draw the second straight line from the point of intersection on the pivot line to the 10,000-ton point on the displacement scale. The point at which the second straight line intersects the GM reduction scale indicates how much reduction in GM has occurred because of free surface effect in the partially flooded compartment. In this example, GM has been reduced 0.2 foot.

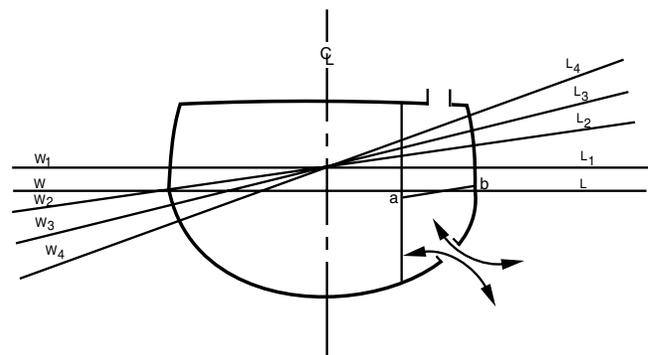
### Free Communication Effect

Thus far, the stability changes caused by the effect of free surface and by the addition of the weight of the flooding water have been considered. In certain instances, it is also necessary to make allowance for stability changes that occur when an off-center compartment is in free communication with the sea.

If a boundary of an off-center compartment is so extensively ruptured that the sea can flow freely in and out as the ship rolls, the FREE COMMUNICATION EFFECT will cause a reduction in GM and in GZ. Note that the free communication effect on stability is IN ADDITION TO the effect of free surface and the effect of added weight. To understand the free communication effect, consider an off-center compartment partially full of water and in free communication with the sea, as shown in figure 12-31. (Note that this compartment is free to vent at the top.)

Before the hull is ruptured, the ship floats on an even keel at waterline WL. Then the compartment is partially flooded and left in free communication with the sea. Assume that the water enters the compartment instantaneously (up to the level of the ship's original waterline WL) and is instantaneously frozen solid. If the weight of the frozen water is distributed equally

about the ship's centerline, the ship will undergo parallel sinkage to a new waterline such as  $W_1L_1$ . Since the weight is off center, however, the ship assumes an inclined position with a waterline similar to  $W_2L_2$ .



DCf1231

Figure 12-31. Free communication effect in off-center compartment.

If the water in the compartment is now returned to its fluid state, it will have a waterline a-b that is parallel to (but below) the ship's waterline  $W_2L_2$ . Immediately, however, additional water will flow in from the sea and flood the compartment to the actual level of the ship's waterline  $W_2L_2$ . The ship will therefore sink deeper in the water and will assume a greater list; the waterline will reach a position such as  $W_3L_3$ . Again, additional water will flow in from the sea and flood the compartment to the level of the ship's waterline  $W_3L_3$ ; this will cause the ship to sink even deeper in the water and to assume an even greater list. These interactions will continue until the waterline is at the position represented by  $W_4L_4$ .

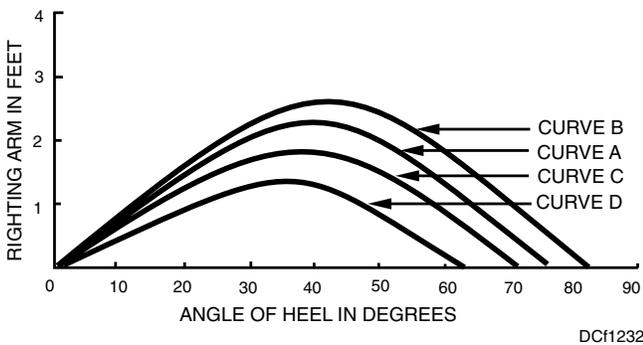
Note that stability is not usually reduced by free communication if the compartment is symmetrical about the ship's centerline. Under certain circumstances, free communication in a centerline compartment may increase the free surface effect, and thereby reduce stability. However, it is important to remember that this reduction in stability occurs from the increased free surface effect, rather than from any free communication effect.

### Summary of Effects of Loose Water

The addition of loose water to a ship alters the stability characteristics by means of three effects that must be considered separately: (1) the effect of added

weight, (2) the effect of free surface, and (3) the effect of free communication.

Figure 12-32 shows the development of a stability curve with corrections for added weight, free surface, and free communication. Curve A is the ship's original stability curve before flooding. Curve B represents the situation after flooding; this curve shows the effect of added weight (increased stability) but it does NOT show the effects of free surface or of free communication. Curve C is curve B corrected for free surface effect only. Curve D is curve B corrected for both free surface effect and free communication effect. Curve D, therefore, is the final stability curve; it incorporates corrections for all three effects of loose water.



**Figure 12-32. Development of stability curve corrected for effects of added weight, free surface, and free communication.**

## LONGITUDINAL STABILITY

Thus far in studying stability, you have been concerned only with TRANSVERSE STABILITY and with TRANSVERSE INCLINATIONS. LONGITUDINAL STABILITY and LONGITUDINAL INCLINATIONS, or TRIM, should also be considered.

Trim is measured by the difference between the forward draft and the after draft. When the after draft is greater than the forward draft, the ship is said to be TRIMMED BY THE STERN. When the forward draft is greater than the after draft, the ship is said to be TRIMMED BY THE BOW or TRIMMED BY THE HEAD. As a ship trims, it inclines about an athwartship axis that passes through a point known as the CENTER OF FLOTATION (CF).

The mean draft that is used to enter the draft scale to read a displacement curve is the draft amidships.

When a ship has trim, however, neither the draft amidships nor the average of the forward and after drafts will give a true mean draft. For most types of ships, the curves of form may be used without correction for trim, PROVIDED the trim is less than about 1 percent of the length of the ship. When the trim is greater, however, the readings obtained from the curves of form must be corrected for trim.

Longitudinal stability is the tendency of a ship to resist a change in trim. The longitudinal metacentric height multiplied by the displacement is taken as a measure of INITIAL longitudinal stability when trim is very small. (It is important to note that the longitudinal metacenter (M1) is NOT the same as the transverse metacenter.) A more accurate measure of the ship's ability to resist a change of trim is made in terms of the moment required to produce a change in trim of a definite amount. The MOMENT TO CHANGE TRIM 1 INCH (MTI) is used as the standard measure of resistance to longitudinal inclination.

---

## REVIEW QUESTIONS

- Q7. The ship's center of gravity is the point at which all weights of the ship may be considered to be concentrated. The force of gravity is considered as acting straight downward, through the center of gravity, at right angles to the waterline.
1. True
  2. False
- Q8. Detailed information concerning changes in the center of gravity of a ship can be obtained from which of the following *NSTM*s?
1. *NSTM*, chapter 096
  2. *NSTM*, chapter 040
  3. *NSTM*, chapter 033
  4. *NSTM*, chapter 010
- Q9. Which of the following terms is used to describe water or other liquid that has a free surface?
1. *Reserve buoyancy*
  2. *Reserve ballast*
  3. *Draft*
  4. *Loose water*

## SUMMARY

Q10. What effect does pocketing have on stability?

1. Improves stability
2. Improves righting arms
3. Improves buoyancy
4. Improves righting moments

Q11. Which of the following effects does NOT alter the stability characteristics of a ship when you have loose water?

1. Added weight
2. Free surface
3. Free communication
4. Added buoyancy

Q12. What does longitudinal stability resist?

1. Change in trim
2. Change in list
3. Draft
4. Heeling

This chapter has introduced you to the terminology used for ship stability; the laws of physics and trigonometry used to determine stability and buoyancy of a ship; and the effects of buoyancy, gravity, and weight shifts on ship stability. Other aspects involved in the study of stability are taken into consideration when an inclining experiment is being conducted, when the ship is being dry docked, or when a grounding has occurred.

Aboard certain ships you may qualify as ballasting officer and be actively involved in maintaining stability. Remember that additional information on this topic may be found in the following publications: *Naval Ships' Technical Manual (NSTM)*, chapter 079, volume 1, and chapter 096; nonresident training courses (NRTCs): *Mathematics*, volume 1; *Mathematics*, volume 2A; *Fireman*; and *Basic Machines*.

## REVIEW ANSWERS

- A1. Detailed information on the laws of mathematics and physics used to determine the buoyancy and stability of a ship are provided in *Naval Ships' Technical Manual (NSTM)*, chapter 079, volume 1, and in *NSTM*, chapter 096. **(1) True**
- A2. Which of the following trigonometric functions is NOT used for making calculations to determine a ship's stability? **(4) Cotangent**
- A3. Which of the following terms best defines force multiplied by the distance from an axis about which you want to find its effect? **(1) Moment of force**
- A4. The volume of water that is moved by the hull of a ship is known as "displacement." **(1) True**
- A5. What measurement is known by the term *freeboard*? **(2) Distance from the waterline to the weather deck edge**
- A6. Which of the following information is NOT contained in the "booklet of inclining experiment data?" **(3) Reserve buoyancy**
- A7. The ship's center of gravity, is the point at which all weights of the ship may be considered to be concentrated. The force of gravity is considered as acting straight downward, through the center of gravity, at right angles to the waterline. **(1) True**
- A8. Detailed information concerning changes in the center of gravity of a ship can be obtained from which of the following *NSTMs*? **(1) NSTM, chapter 096**
- A9. Which of the following terms is used to describe water or other liquid that has a free surface? **(4) Loose water**
- A10. What effect does pocketing have on stability? **(1) Improves stability**
- A11. Which of the following effects does NOT alter the stability characteristics of a ship when you have loose water? **(4) Added Buoyancy**
- A12. What does longitudinal stability resist? **(1) Change in trim**

